MODELING NOISE FOR A BETTER SIMPLIFICATION OF SKELETONS

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ABSTRACT

The skeleton of an object is the locus of the centers of maximal discs included in the shape. The skeleton provides a compact representation of objects, useful for shape description and recognition. A well-known drawback of the skeleton transformation is its lack of continuity. This paper is concerned with the modeling of noise that may affect objects and the concequence of this noise on the skeleton. A graph (called the parameter graph) is introduced, on which branches due to noise are characterized. We deduce from this preliminary study a method to simplify skeletons. It depends on thresholds that can be chosen directly on the parameter graph associated to each skeleton.

1. INTRODUCTION

The skeleton of an object is a thin figure, centered in the shape and which summarizes its general form [1]. It provides a synthetic representation of objects, useful in image analysis for shape description.

A drawback of the skeleton transformation is its lack of continuity. Noise on the boundary of an object may significantly change the aspect of its skeleton. A simplification algorithm is therefore necessary to remove peripherical branches having no perceptual relevance.

The methods for simplification are generally based on the same general scheme. Peripherical branches are shortened by removing end points one after the other while they verify a *removing criterion*. By construction, the simplified skeleton is a subset of the initial skeleton having the same class of homotopy.

Different removing criteria have been proposed. One can for instance measure the difference between the initial shape and the shape reconstructed from the simplified skeleton. Branches are shortened as long as this difference remains smaller than a fixed threshold [2, 3]. More complex criteria may be found in [4].

Existing methods left unresolved some crucial aspect of the simplification problem. They do not study the effect of noise on the skeleton. They depend on thresholds that are difficult to find automatically as they change with the objects.

In this paper, we raise three questions:

1. *What type of noise may affect real objects ?* A model of noise is proposed that turns out to be realistic for a large amount of objects.

2. *What is the influence of this type of noise on the skeleton ?* The effect of noise on the skeleton is studied and a characterization of noisy branches is deduced.

3. *How to select parameters for the simplification ?* A graph is introduced on which the parameters can directly be selected.

Section 2 recalls the definition of the skeleton and describes a method to compute it. In section 3, a model of noise is introduced and its effect on the skeleton is studied. Section 4 proposes a simplification method.

2. COMPUTING THE SKELETON

By definition, the skeleton $Sk(X)$ of an object X is the locus of the centers of the maximal balls of X. A ball B included in X is said to be maximal if there exists no other ball included in X and containing B (Fig. 1).

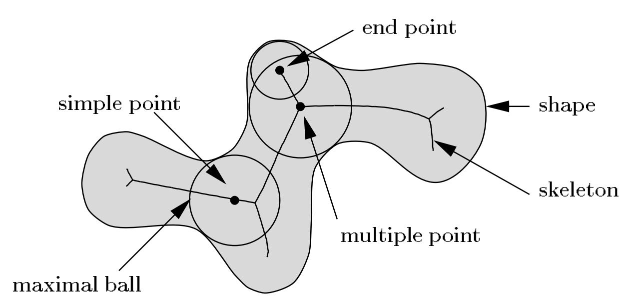

Figure 1: Vocabulary relative to skeletons.

Numerous methods have been proposed to extract the skeleton. They can be classified in two main families: *discrete methods* and *continuous methods*.

Discrete methods work directly on binary images. The skeleton is a set of pixels that is computed using distance transforms or morphological thinnings [5, 6].

Continuous methods are derived from computational geometry. They are generally based on the computation of the Voronoi graph of a set of points located on the boundary of the object [2, 7, 8].

In this paper, the skeleton is computed, using the continuous approach described in [2]. The input is a set of points $\{p_i\}_{i=1}^n$ located on the boundary of a smooth object X. The first step of the method consists in computing the Voronoi graph of the boundary points p_i. The skeleton is defined as the set of the Voronoi edges and vertices that are completely included in X (Fig 2).

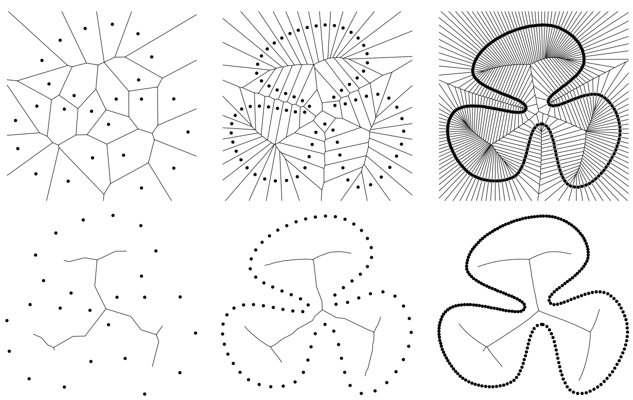

Figure 2: Voronoi graphs and approximated skeletons.

3. MODELING NOISE

Presence of parasite branches on the skeleton is generally explained by presence of noise on the boundary of the object. It is therefore logical to model the type of noise one would like to remove. In the following, we consider that the shape X to skeletonize is sampled by a set of points $\{q_i\}_{i=1}^n$ where each point q_i can be written as $q_i = p_i + e_i$. The points p_i belong to the boundary of X and the vectors e_i represent the added noise around points. The skeleton is approached using the Voronoi graph of the noisy points q_i.

In order to explain the influence of this type of noise on the computed skeleton, we introduce two preliminary definitions:

Definition 1 (Thickness) *Let X be a continuous shape and $Sk(X)$ the skeleton of X. The thickness $\rho(s)$ at a point s of the skeleton is the radius of the maximal ball centered on s.*

Definition 2 (Bisector angle) *Let X be a continuous shape and $Sk(X)$ the skeleton of X. Let s be a simple point of the skeleton. The maximal ball centered on s touches the boundary of X at two contact points p_0 and p_1. The bisector angle $\alpha(s)$ is the angle $\widehat{p_0 s p_1}$ lying between 0 and π. If s is not a simple point but a terminal point or an end point, the bisector angle $\alpha(s)$ is computed by passing to the limit.*

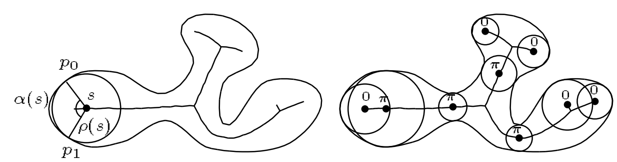

Figure 3: Bisector angle and thickness.

The bisector angle has remarkable properties [9]. It equals π where the thickness is extremum and 0 at the end points of the skeleton (Fig. 3).

In practice, if s designates a Voronoi vertex of the approximated skeleton and $[p_0 p_1 p_2]$ its associated Delaunay triangle, the thickness $\rho(s)$ and the bisector angle $\alpha(s)$ can be approximated by:

$$\begin{aligned} \rho(s) &= d(s, p_0) = d(s, p_1) = d(s, p_2) \\ \alpha(s) &= max(\widehat{p_0 s p_1}, \widehat{p_1 s p_2}, \widehat{p_2 s p_0}) \end{aligned}$$

In order to visualize the effect of noise, one can represent the vertices s of the skeleton on a graph entitled the *parameter graph* in which vertices are plotted according to $\rho(s)$ against $\alpha(s)$ instead of their classical Cartesian coordinates. Each vertex s of the skeleton is associated with a point having coordinates $(\alpha(s), \rho(s))$ in the parameter graph.

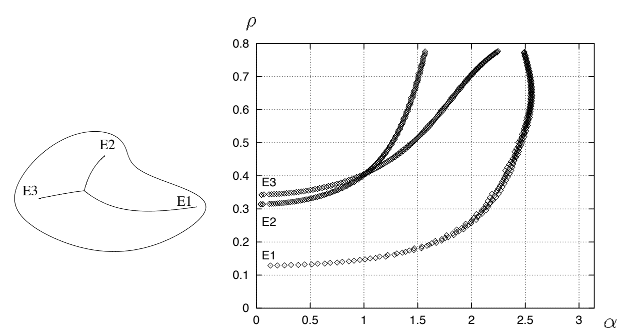

Figure 4: Skeleton of a synthetic object and its parameter graph.

When there is no noise (synthetic object of Fig. 4), the skeleton structures are easily identified within the parameter graph. End points lie on the straight-line ($\alpha = 0$) and branches are represented by curves.

When noise is added to the shape boundary (Fig. 5), the aspect of the parameter graph changes. Indeed, most of the vertices that were initially lying on the branches of the skeleton are now scattered next to the bottom left of the parameter graph plotting an hyperbolic form.

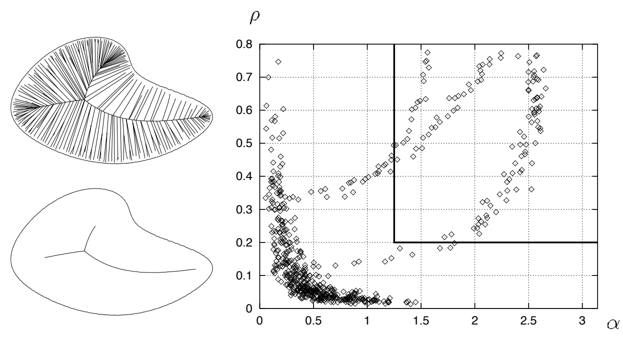

Figure 5: Skeleton of a noisy object, simplified skeleton and the parameter graph.

The same phenomenon occurs with objects obtained from binary images and prove the relevance of our noise model. However, in the latter case, points organize themselves in parallel strata (Fig. 7).

It is possible to explain the noise induced hyperbolic form. Let s be a vertex of the skeleton, a and b the two points of the boundary such that $\alpha(s) = \widehat{asb}$ and $\rho(s) = d(a, s) = d(b, s)$. The set of vertices s generated by two boundary points a and b distant from d has equation:

$$\rho = \frac{d}{2 \sin \frac{\alpha}{2}}$$

If one plots ρ as a function of α with a fixed value for d on the parameter graph, a curve is obtained which perfectly fits the hyperbolic form (Fig. 6). With objects obtained from binary images, the distance d between two boundary points is quantified (owing to the discrete representation of binary images). The presence of strata in the hyperbolic form confirms the existence of forbidden values for d.

Thus, the noise disturbs neighbourhood relationships in the Voronoi graph. Boundary points that are distant from d generate parasite branches on the skeleton. When noise remains small, d is also small. Perturbations due to noise stay local.

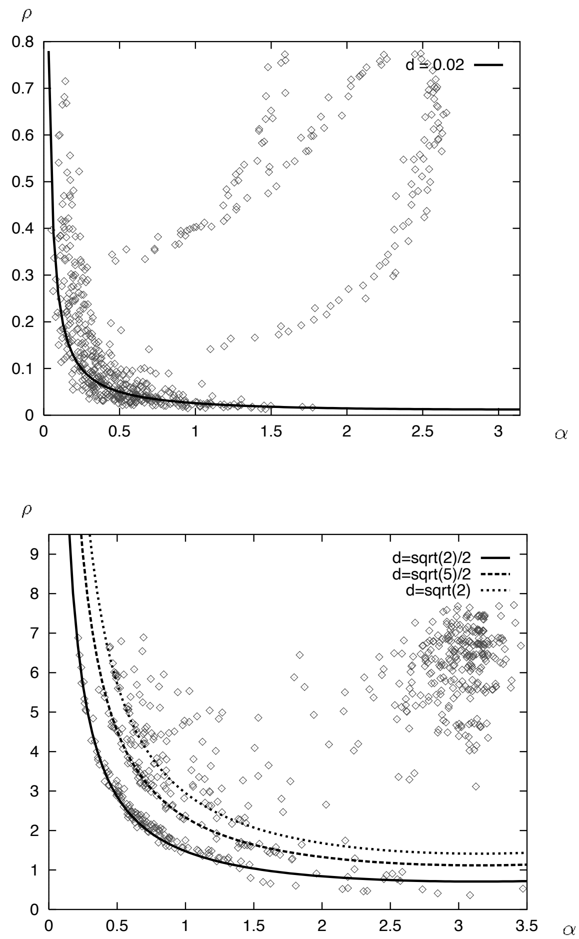

Figure 6: Modeling noise with $\rho = \frac{d}{2 \sin \frac{\alpha}{2}}$.

4. PROPOSED CRITERION

A result of the previous section is that noisy vertices of the skeleton are charaterized by a small bisector angle or a small thickness. Consequently, we propose to remove an end point s of the current skeleton if the two parameters $\alpha(s)$ and $\rho(s)$ verify:

$$(\alpha(s) < \alpha_0) \; or \; (\rho(s) < \rho_0)$$

The proposed criterion depends on two thresholds α_0 and ρ_0. The first one, α_0 controls the lost of information. If α_0 equals 0, the skeleton is never simplified. The second threshold ρ_0 gives an indication of the size of the object.

One can choose for instance $\alpha_0 = \frac{2\pi}{5}$ and $\rho_0 = 5$ for objects obtained from binary images. But, the two thresholds can also be selected directly on the parameter graph. Indeed, in this representation, our simplification method consists in removing end points located below the thick line plotted on the parameter graphs (Fig. 5 and 7). This line cut off the hypebolic form from the rest of the graph.

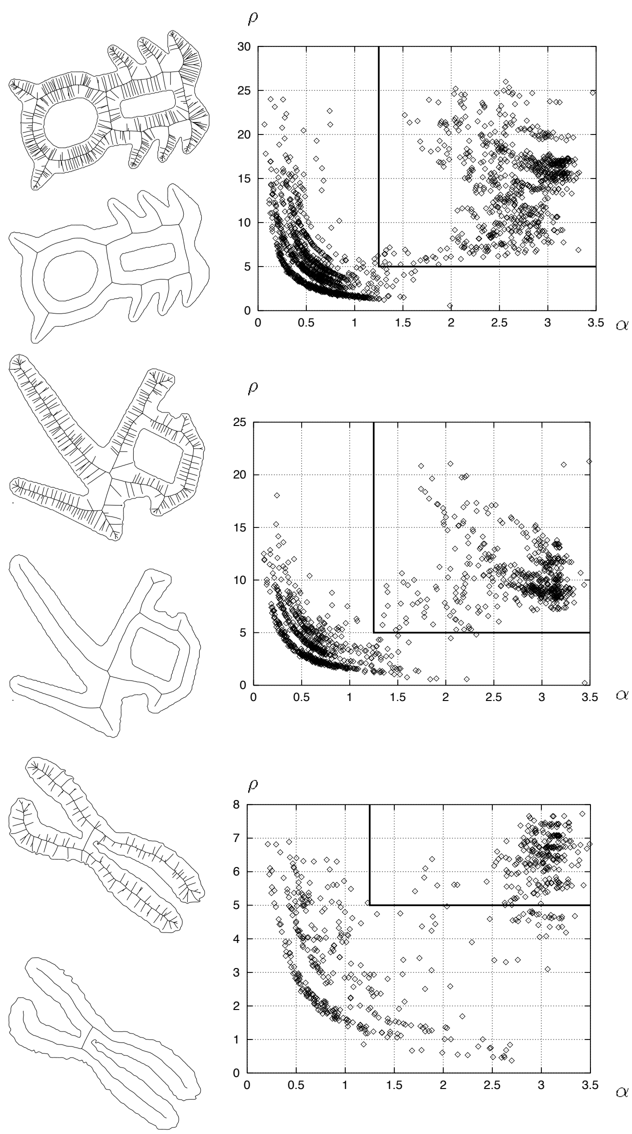

Figure 7: Skeletons of binary objects, simplified skeletons and parameter graphs.

5. CONCLUSION

In this paper, we analyse the influence of noise on the skeleton. We deduce a characterization of noisy branches and a simplification method for continuous skeletons. The study of other noise models could be the subject of further works.

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