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Using Polyballs to Approximate Shapes and Skeletons

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Abstract

This paper presents an approach to approximate the skeleton of continuous shapes either in 2D or 3D space. The data required is a sampling of the boundary of the shape.

We call polyball any finite union of balls. A preliminary work on polyballs shows that their skeletons consist of simple components (line segments in 2D and polygons in 3D). To construct these components, only the computation of a Voronoi graph is required.

Recent papers have proposed to approximate the skeleton of continuous shapes using the Voronoi graph of boundary points. An original reformulation of these methods is presented here, using polyballs. It allows to build a hierarchy of simplified skeletons.

An application in the frame of an European project in the field of medicine and biology is also presented. The skeleton by influence zones is computed in real time, which validates our approach.

1 Introduction

The notion of skeleton was first introduced by Blum [4]. The skeleton of an object is a thin figure centered in the shape and which summarizes its general form. It can be equivalent to the boundary representation but allows an easier description of the shape.

Numerous methods have been proposed in order to extract the skeleton. They can be classified in three main families:

• Discrete methods. The shape is transmited to us through a binary image. The skeleton is redefined in the discrete space, and only discrete objects are handled [1, 7, 12, 15].

• Semi-continuous methods. The shape is discerned to us through a sampling of its boundary. The skeleton is approached by taking a subgraph of the Voronoi graph of the sampling boundary points [5, 6, 10].

• Exact methods. Unlike the other methods, the continuous shape is known and the exact skeleton is searched. So far, this problem has been solved for very few objects (polygons [9], ellipses)

This work concerns the two last points. Firstly, the problem of computing the exact skeleton is solved for a new subset of the $2D$ and $3D$ continuous shapes, called polyballs. Secondly, we propose an original way to reformulate semi-continuous methods. Finally, we propose to use skeletons for shape description.

1.1 Notations and definitions

$\mathbb{R}^n$ designates the Euclidean n-dimensional space and d the Euclidean distance. For any set of points $X \in \mathbb{R}^n$, ∂X denotes its boundary and X^c its complement.

The skeleton $Sk(X)$ of an object $X \in \mathbb{R}^n$ is the locus of the centers of the maximal balls of X. A ball B included in X is said to be maximal if there exists no other ball included in X and containing B.

2 Polyballs

Computing the exact skeleton of a continuous shape is a complex problem. In $2D$ space, the solution is known only for some simple geometrical shapes. Lee [9] has proved that the exact skeleton of a polygon was made up of straight-line segments and portions of parabolic curves. In $3D$, the skeleton of a polyhedron consists of pieces of quadrics [3].

In this section, the problem of finding the exact skeleton is solved for a new subset of continuous shapes: the polyballs.

2.1 Polyballs and generating balls

By analogy with the polygons, we call polyball an object obtained by a finite union of balls. Let Y be a polyball of the nD space: $Y = \bigcup_{i\in\{0,\ldots,n\}} B_i$.

Note that Y is not necessarily simply connected, nor connected. The balls B_i are called generating balls. Among the generating balls, it will be useful to distinguish those having particular intersections with the others. In order to simplify the classification, generating balls are assumed not to be tangent.

Definition 1 *In 2D or 3D, a generating ball B_i is said to be disconnected if it intersects no other generating ball (Figures 1 and 2).*

Definition 2 *In 3D, the generating balls B_i and B_j are said to be quasi-disconnected if their intersection $B_i \cap B_j \neq \emptyset$ intersects no other generating ball.*

2.2 Skeletons of 2D and 3D polyballs

In $2D$ as in $3D$ space, the boundary of Y has particular points, in finite number, located at the intersection of two or more generating balls. These points, referred to as singular points, are characterized as follows:

Definition 3 *In $2D$, p is a singular point of Y* $\iff$ $\exists B_i, B_j \;\; p \in B_i \cap B_j \cap \partial Y$

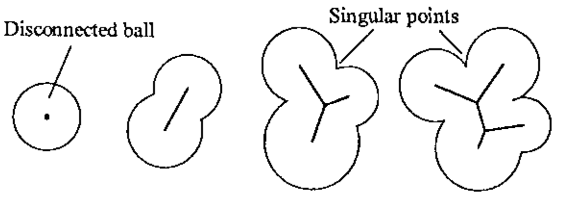

Figure 1: Skeletons of 2D polyballs.

Definition 4 *In $3D$, p is a singular point of Y* $\iff$ $\exists B_i, B_j, B_k \;\; p \in B_i \cap B_j \cap B_k \cap \partial Y$

The two theorems below provide a constructive description of $2D$ and $3D$ skeletons using disconnected balls, quasi-disconnected balls and singular points. Simple examples of polyballs and skeletons are provided in figures 1 and 2.

Theorem 1 *In $2D$, the skeleton of Y is made up of the centers of the disconnected balls and a subset of the Voronoi graph of its singular points. This subset is formed of the straight-line segments included in Y.*

Theorem 2 *In $3D$, the skeleton of Y is made up of the centers of the disconnected balls, the straight-line segments connecting the centers of the two by two quasi-disconnected balls and a subset of the Voronoi graph of its singular points. This subset is formed of the straight-line segments and polygons which are included in Y.*

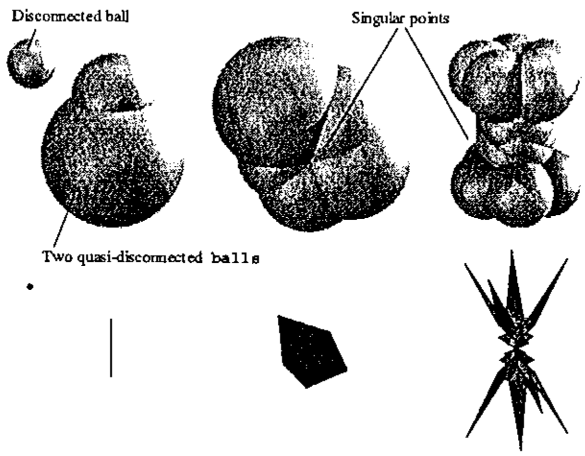

Figure 2: $3D$ simple polyballs and their skeletons.

Figure 3: A continuous shape, a sampling of its boundary and the corresponding approached skeleton.

To conclude, the skeleton of a polyball is made up of simple components as line segments in $2D$ and polygons in $3D$. Furthermore, its computation comes down to the computation of the Voronoi graph which is a very famous problem of computational geometry [13] and for which exist efficient algorithms [3].

Thus, polyballs seem to be very interesting objects to approximate continuous shapes. Such spherical representations are useful for graphical display [11] and modelisation of molecular surfaces [8].

3 Continuous shapes

In this section, we are considering equally $2D$ and $3D$ shapes. The possibility to use polyballs to approach continuous shapes is first discussed. Then, the question whether the skeleton of a polyball can approximate the skeleton of a continuous shape is raised.

3.1 Approximating continuous shapes

Let X be a continuous shape of $\mathbb{R}^n$ and E_w a set of points which sample its boundary. In order to measure the quality of the sampling, we define the sampling density, denoted w as the greatest number such that: $\forall x \in \partial X,\ \exists e \in E_w,\ d(x,e) < w^{-1}$

We are looking for a polyball which approaches X. To do so, the Voronoi graph of E_w is first computed. A collection of balls, circumscribed about the Delaunay simplexes, can be derived from this computation. Let Y_w be the polyball formed by the above mentioned balls whose centers belong to X. The convergence of Y_w to X is ensured if X is r-regular. A proof in $2D$ space may be found in [6].

3.2 Approximating continuous skeletons

Assuming X to be regular enough (the boundary is C^3), Schmitt [14] has demonstrated that Delaunay balls tend to maximal balls, and thus, the skeleton of Y_w tends to the skeleton of X.

Note that the polyball Y_w and its skeleton can easily be derived from the Voronoi graph of the sampling points. The complexity is thus $O(n.\log n)$ in $\mathbb{R}^2$ and $O(n^2)$ in the worst case in $\mathbb{R}^3$.

Figure 3 illustrates our skeletonization method in $3D$ space. There were about 2000 sampling

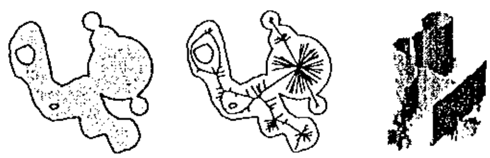

Figure 4: A $2D$ shape, its approached skeleton and the corresponding piling of simplified skeletons.

points. The computation takes 5 secondes on an INDIGO RS4000 Silicon Graphics workstation. The approached skeleton has good properties: it is homotopic to X, rotation invariant and thin. But, it is also sensitive to small boundary disturbances.

A method to simplify noisy $2D$ and $3D$ skeletons may be found in [2]. The extremities of the skeleton (straight line segments in $2D$ and polygons in $3D$) are sequentially removed according to an angular criterion. The piling of the different simplified skeletons leads to a hierarchical representation useful for shape description and pattern matching (Figure 4).

4 Application

Our work has already been applied in the frame of an European project for biology and medecine (AIM program, IMPACT project No A2017 92-93).

Pathologists study histological samples under a microscope, including different structures such as cells and glands (Figure 5). The local distribution of cells in the intercellular liquid makes it possible to grade the state of advancement of a pathology (hyperplasia, cancer). To this end, it is necessary to assign each gland a zone of influence, in which a sub-population of cells may be quantified and studied.

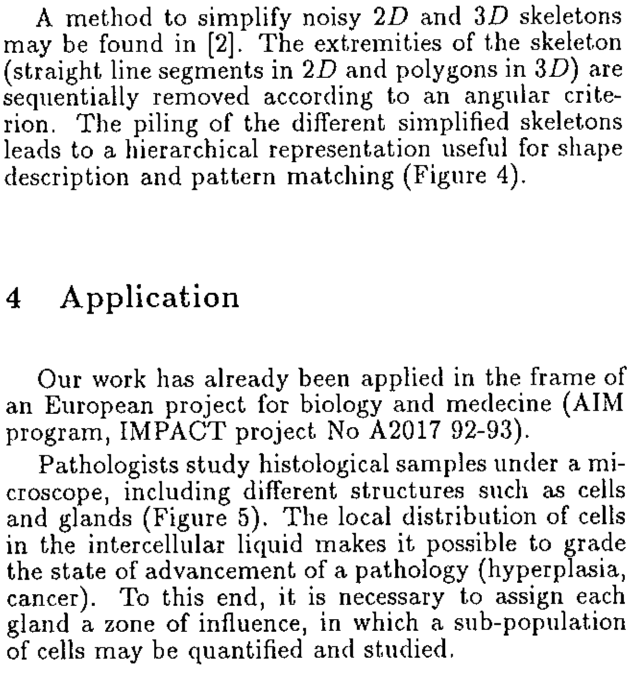

Figure 5: User interface facilities and Skiz.

Finally, the problem is to compute in $2D$ space the skeleton by influence zones, denoted Skiz, of a set of glands approximated by a set of polygons. As the Skiz is included in the skeleton of X^c, its computation is a particular case of our previous work and is done in real time on a compatible 486 PC (see Figure 5).

5 Conclusion

New objects called polyballs have been studied. The skeleton of a polyball is proved to have simple components, which allows its exact computation either in $2D$ or $3D$ space. For this reason, polyballs are very interesting objects to approximate continuous shapes. If correctly simplified, $3D$ skeletons are promised to be useful for analysing $3D$ objects.

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